

Section 6.4 Solving Absolute Value Equations and Inequalities

Solving Absolute Equations

Recall that an absolute value is the distance from the zero on the number line. Therefore, $|x| = 7$ has two solutions either $x = 7$ or $x = -7$. We can use this knowledge to write a procedure to solve more complicated absolute value equations. The steps follow:

How to solve absolute value equations:

1. Isolate the absolute value sign. Remember you can't do anything inside it until you get rid of it.
2. To get rid of the absolute value you split the problem into the positive and the negative side.
3. Solve each equation separately.

Example 1: Solve $|3x + 7| + 3 = 8$

$$|3x + 7| = 5$$

Isolate the absolute value sign by subtracting 3 from each side.

| | |
|--------------|---------------|
| Positive | Negative |
| $3x + 7 = 5$ | $3x + 7 = -5$ |

Split into the positive and negative sides to get rid of the absolute value.

| | |
|------------|------------|
| $3x = -2$ | $3x = -12$ |
| $x = -2/3$ | $x = -4$ |

Solve each equation.

Solution: $x = -2/3$ or $x = -4$

Solving Absolute Value Inequalities

How to solve absolute value inequalities:

1. Isolate the absolute value
2. Split into the positive and negative side and remember to flip the inequality for the negative side.
3. Solve each inequality separately.
4. If it can be written as a 'between' solution, do it.

Example 2: solve $|2x + 4| - 7 = 3$

$$|2x + 4| = 10$$

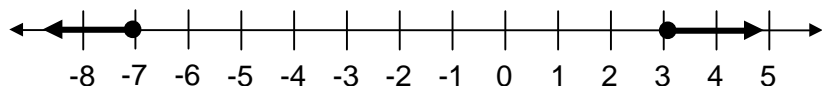
| | |
|---------------|----------------|
| Positive | Negative |
| $2x + 4 = 10$ | $2x + 4 = -10$ |
| $2x = 6$ | $2x = -14$ |
| $x = 3$ | $x = -7$ |

Isolate the absolute value sign by adding 7 to each side.

Split into the positive and negative sides and flip the negative sign.

Solve each inequality.

This solution is not a between solution.



Solution: $x = 3$ or $x = -7$

Example 3: $|2x + 3| + 2 = 7$

$$|2x + 3| = 5$$

Isolate the absolute value

Positive

$$2x + 3 = 5$$

Negative

$$2x + 3 = -5$$

Split into positive and negative

$$2x = 2$$

$$x = 1$$

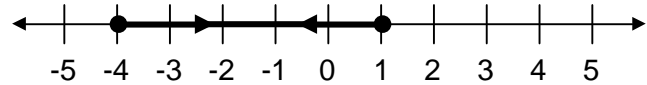
$$2x = -8$$

$$x = -4$$

Solve each

Graph:

This is a between (and) so it should be written as such.



Solution: $-4 \leq x \leq 1$