

Show your work using Calculus where appropriate!

1. Determine the limit of the given function

a.  $\lim_{x \rightarrow -3} (2x^2 - 3x + 2)$

$$2(9) + 6 + 2 = \boxed{26}$$

b.  $\lim_{x \rightarrow -2} \left( \frac{x^2 + 3x + 2}{(x+2)^2} \right) = \lim_{x \rightarrow -2} \frac{\cancel{(x+2)}(x+1)}{\cancel{(x+2)}^2}$

D.N.E  
can't remove  
the discontinuity

c.  $\lim_{x \rightarrow 0} \left( \frac{3^x - 4^x}{3^x + 4^x} \right) = \frac{1 - 1}{1 + 1} = \boxed{0}$

d.  $\lim_{x \rightarrow 0} \left( \frac{2x^4 - 6x^3 - 3x - 12}{-5x^4 + 4x^3 + x^2 - 2x} \right) = \boxed{-\frac{2}{5}}$

change to  $\infty$

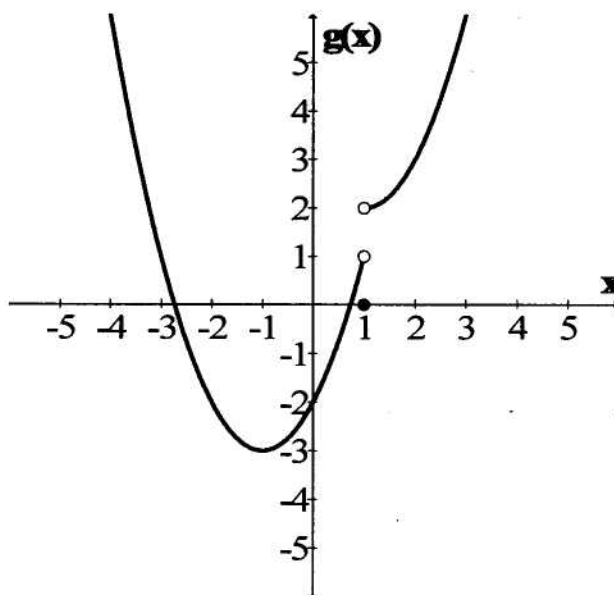
2. Use the graph below to answer the following questions.

a.  $\lim_{x \rightarrow 1^-} g(x)$   
 $\boxed{1}$

b.  $\lim_{x \rightarrow 1^+} g(x)$   
 $\boxed{2}$

c.  $\lim_{x \rightarrow 1} g(x)$   
 D.N.E.

d.  $g(1) = \boxed{0}$



3. Find all numbers for which the function is not continuous. At each point(s) of discontinuity determine if the discontinuity is jump, infinite, or removable.

a.  $f(x) = \frac{5}{x^2 - 5x - 14} = \frac{5}{(x-7)(x+2)}$

Non-Removable, infinite discontinuities  
 at  $x=7$  and  $x=-2$ .

b.  $f(x) = \frac{x^2 + 5x + 6}{x^2 + x - 2} = \frac{(x+3)(x+2)}{(x+2)(x-1)}$

Nonremovable  
 discontinuity  
 at  $x=1$   
 Removable discontinuity  
 at  $x=-2$ .

change to

$$x=3$$

4. Given the function  $f(x) = \begin{cases} x^2 - 2, & x < 3 \\ x + 5, & x \geq 3 \end{cases}$

Determine whether the function is continuous at  ~~$x=1$~~   $x=3$ . Show your work.

$$\lim_{x \rightarrow 3^-} = 9 - 2 = 7$$

$$\lim_{x \rightarrow 3^+} = 3 + 5 = 8$$

Not continuous  
b/c lim from  
the left not  
equal to lim  
from the right  
at  $x=3$

The function is not

5. Given the function  $f(x) = \begin{cases} 2x^2 - 3x + 2, & x < 1 \\ 3x + k, & x \geq 1 \end{cases}$

Find the value of  $k$  which makes the function continuous. Show your work.

$$\lim_{x \rightarrow 1^-} f(x) = 2(1)^2 - 3(1) + 2 = 1$$

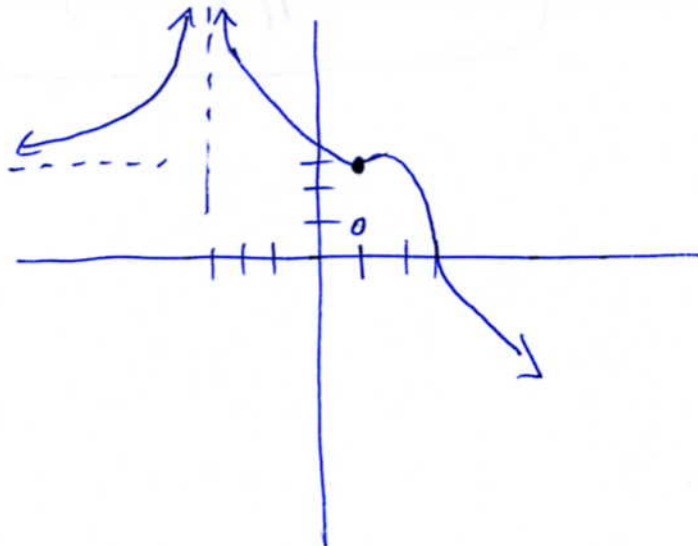
$$\lim_{x \rightarrow 1^+} f(x) = 3(1) + k$$

$$\begin{aligned} 3 + k &= 1 \\ k &= -2 \end{aligned}$$

$k$  must =  $-2$  to

6. Draw a sketch of a function that satisfies all of the following conditions:

$\lim_{x \rightarrow -\infty} f(x) = 3$	$\lim_{x \rightarrow -3^-} f(x) = +\infty$	$\lim_{x \rightarrow 1^+} f(x) = 3$	<del><math>\lim_{x \rightarrow 1^+} f(x) = 1</math></del>
$\lim_{x \rightarrow -3^+} f(x) = +\infty$	$\lim_{x \rightarrow 1^-} f(x) = 1$	$f(1) = 3$	$f(3) = 0$



7. Given the function  $f(x) = 2x^2 + 3x - 5$  at the point  $(2, 9)$

$$f'(x) = 4x + 3$$

$$f'(2) = 8 + 3 = 11$$

- a) Find the slope of the curve at the given point. Show your work that leads to the answer.

$$\begin{aligned} \text{Slope} &= \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \rightarrow 0} \frac{2(2+h)^2 + 3(2+h) - 5 - (2(2)^2 + 3(2) - 5)}{h} \\ &= \lim_{h \rightarrow 0} \frac{\cancel{8} + 8h + 2h^2 + \cancel{6} + 3h - \cancel{5} - \cancel{8} - \cancel{6} + \cancel{5}}{h} \\ &= \lim_{h \rightarrow 0} (8 + 2h + 3) = \boxed{11} \end{aligned}$$

- b) Write an equation for the tangent line at the given point.

$$y - 9 = 11(x - 2)$$

- c) Write an equation for the normal line at the given point.

$$y - 9 = -\frac{1}{11}(x - 2)$$