

Calc BC Chapter 4 Practice Test  
Pg 1 of 6

1. a.  $x=1$
- b.  $x=3$
- c.  $1 < x < 3$
- d.  $x < 1$     $x > 3$
- e.  $-\infty < x < 2$
- f.  $2 < x < \infty$

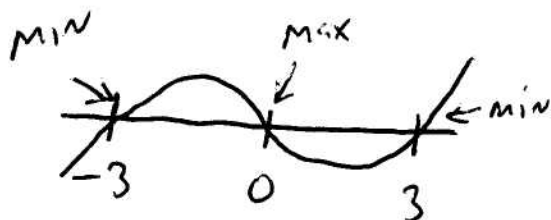
2.  $g(x) = x^4 - 18x^2 + 2$

a.  $g'(x) = 4x^3 - 36x$

$$4x^3 - 36x = 0$$

$$4x(x^2 - 9) = 0$$

$$x=0 \quad x=3 \quad x=-3$$



Relative min    $(-3, -79)$     $(3, -79)$   
Relative max    $(0, 2)$

b.

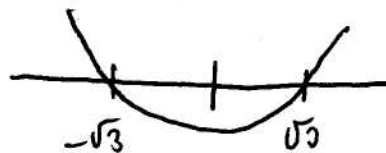
$g(x)$  decreasing    $x < -3$  ,  $0 < x < 3$

c.

$g(x)$  concave up

$$g''(x) = 12x^2 - 36$$

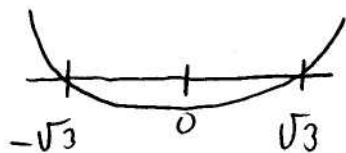
$$12(x - \sqrt{3})(x + \sqrt{3})$$



d.

concave up    $x < -\sqrt{3}$  and  $x > \sqrt{3}$   
concave down    $-\sqrt{3} < x < \sqrt{3}$

e.



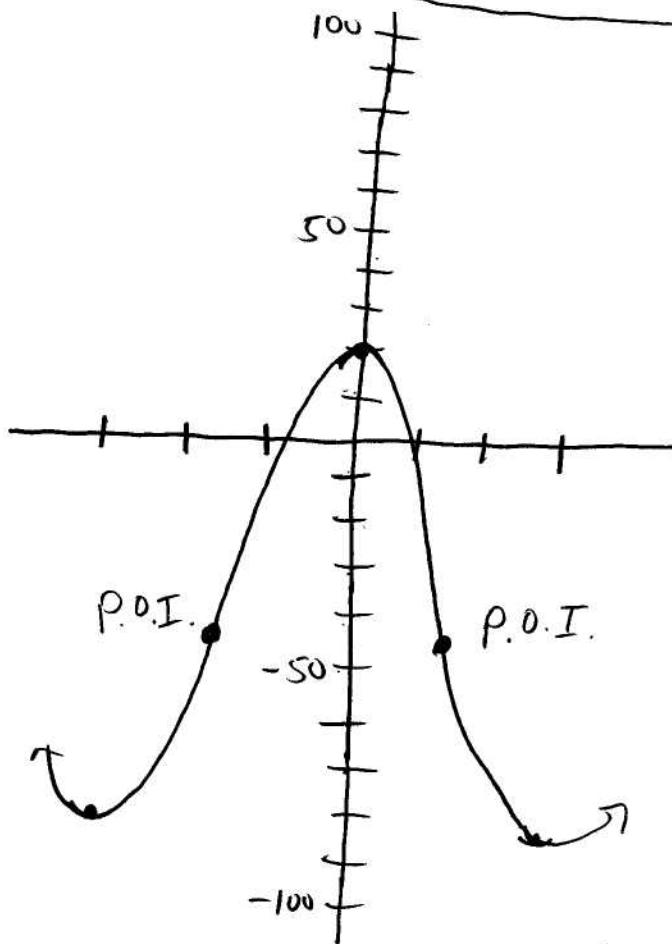
Points of inflection  
at  $x = -\sqrt{3}$ ,  $x = \sqrt{3}$

$$\begin{aligned} g(\sqrt{3}) &= (\sqrt{3})^4 - 18(\sqrt{3})^2 + 2 \\ &= 9 - (18)(3) + 2 = -43 \end{aligned}$$

$$g(-\sqrt{3}) = -43$$

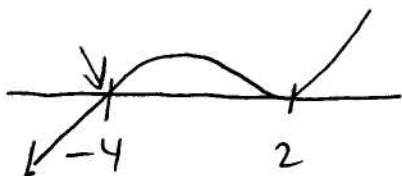
Points of Inflection  $(-\sqrt{3}, -43)$ ,  $(\sqrt{3}, -43)$

f.



$$f'(x) = (x-2)^2(x+4)$$

Also Critical #'s  $x=2$   $x=-4$   
Double Root



a) No relative max b/c  $f'$  never changes from pos to neg

b) relative min at  $x=-4$  b/c  $f'$  changes from neg to pos at  $x=-4$ .

c)  $f''(x) = (x-2)^2(1)(1) + (x+4)(2)(x-2)(1)$  product rule

$$= x^2 - 4x + 4 + (2x+8)(x-2)$$

$$= x^2 - 4x + 4 + 2x^2 - 4x + 8x - 16$$

$$= 3x^2 - 12 = 3(x^2 - 4)$$

$$f''(x) = 3(x-2)(x+2)$$

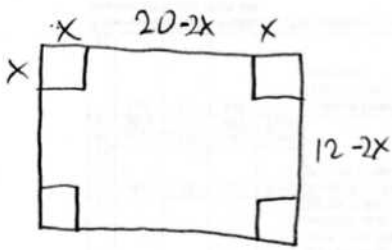
$$x=2 \quad x=-2$$



Root	Add
-12	-4
(-6)(2)	

Points of inflection at  $x=-2$  and  $x=2$   
b/c  $f''$  changes signs at those points.

4.



$$V = x(20-2x)(12-2x)$$

$$V = x(320 - 40x - 24x + 4x^2)$$

$$V = 4x^3 - 64x^2 + 320x$$

$$V' = 12x^2 - 128x + 320$$

$$V' = 4(3x^2 - 32x + 80)$$

$$V' = 4(x-4)(x-10)$$



Max when  $x=4$

	$x-4$	
$3x$	$3x^2$	$12x$
$-10$	$-20x$	$80$
	$\hline$	$\hline$
	$M$	$A$
	$240$	$-32$
	$(3)(80)$	
	$(-4)(-60)$	
	$(-8)(-20)$	
	$(-12)(-20)$	

$$\begin{aligned} \text{Volume} &= 4(20-8)(12-8) = 4(12)(4) \\ &= \boxed{64 \text{ cm}^3} \end{aligned}$$

5.

$$f(x) = (1 + \tan x)^{2/3} \quad f(0) = 1$$

$$f'(x) = \frac{2}{3}(1 + \tan x)^{-1/3} \sec^2 x$$

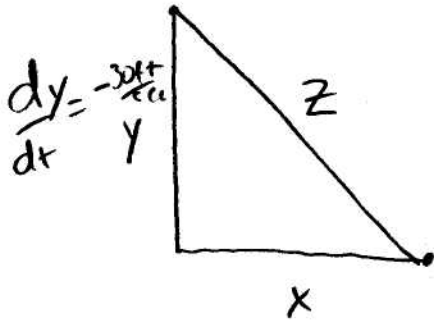
$$f'(x) = \frac{2 \sec^2 x}{3(1 + \tan x)^{1/3}} \quad f'(0) = \frac{2(1)}{3(1+0)^{1/3}} = \frac{2}{3}$$

tangent line  $L(x) = \frac{2}{3}x + 1$

$$\begin{aligned} L(.06) &= \frac{2}{3}(.06) + 1 \\ &= (2)(.02) + 1 = \boxed{1.04} \end{aligned}$$

6.

10 15



$$\frac{dx}{dt} = -40 \frac{\text{ft}}{\text{sec}}$$

$$z^2 = x^2 + y^2$$

Find  $\frac{dz}{dt}$  when

$$x = 150 \quad y = 100 \quad z = \sqrt{32500}$$

$$\frac{dy}{dt} = -30 \frac{\text{ft}}{\text{sec}} \quad \frac{dx}{dt} = -40 \frac{\text{ft}}{\text{sec}}$$

$$2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{z} \left[ x \frac{dx}{dt} + y \frac{dy}{dt} \right]$$

$$\frac{dz}{dt} = \frac{1}{\sqrt{32500} \text{ft}} \left[ 150 \text{ft} \left( -30 \frac{\text{ft}}{\text{sec}} \right) + 100 \text{ft} \left( -40 \frac{\text{ft}}{\text{sec}} \right) \right]$$

$$= \frac{-8500}{\sqrt{32500}} = -47.150 \frac{\text{ft}}{\text{sec}}$$

The distance is changing at a rate of  
 $47.150 \frac{\text{ft}}{\text{sec}}$

