Calc BC Chapter 8 Practice Test

1.

$$\lim_{x\to1} \frac{x^2+4x-5}{x^3-5x^2+4x} \quad \frac{0}{0}$$

$$\lim_{x\to1} \frac{2x+4}{3x^2-10x} = \boxed{\frac{6}{7}}$$

2.

$$\lim_{x\to\infty} \left(e^{3x}+4x\right)^{2/x} \qquad f(x) = e^{\ln f(x)}$$

$$\ln f(x) = \frac{2}{x}\ln\left(e^{3x}+4x\right)$$

$$\lim_{x\to\infty} \left(\frac{2\ln\left(e^{3x}+4x\right)}{x}\right) = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2(3e^{3x}+4)}{e^{3x}+4x} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2(9e^{3x})}{3e^{3x}+4} = \frac{\infty}{\infty}$$

$$\lim_{x\to\infty} \frac{2(\cancel{27})e^{3x}}{\cancel{9}e^{3x}} = 6$$

$$f(x) = e^{\ln(f(x))} = \boxed{e^6}$$

3.

$$\lim_{x\to\infty} \left(\frac{(\ln x)^3}{4x^2}\right) = \frac{\infty}{\infty} \quad \lim_{x\to\infty} \frac{3(\ln x)^2}{8x^2} \quad \frac{\infty}{\infty} = \frac{6\ln x}{16x^2} \quad \frac{\infty}{\infty} \quad \lim_{x\to\infty} \frac{6}{32x^2} = 0$$

so $4x^2$ grows faster

$$\lim_{x\to\infty} \frac{10^x}{e^{2x}} = \lim_{x\to\infty} \left(\frac{10}{e^2}\right)^x = \infty$$ so 10^x grows faster

$(\ln x)^3,\ 4x^2,\ e^{2x},\ 10^x$

4. $\int_3^4 \frac{dx}{x^2-9}$ an infinite discontinuity at $x=3$

$\lim_{b\to 3^+} \int_b^4 \frac{dx}{x^2-9}$

$\frac{1}{x^2-9} = \frac{A}{x-3} + \frac{B}{x+3}$

$A(x+3) + B(x-3) = 1$

$x=-3 \quad B = -\frac{1}{6}$

$x=3 \quad A=\frac{1}{6}$

$\lim_{b\to 3^+} \int_b^4 \left(\frac{1}{6(x-3)} - \frac{1}{6(x+3)}\right) dx =$

$\lim_{b\to 3^+} \left(\ln\left[\frac{(x-3)}{(x+3)}\right]_0^b \right) = \ln(0)$ $\boxed{\text{diverges}}$

5. $\int_1^\infty \frac{4+\cos(3x)}{x^4} = \lim_{b\to\infty} \int_1^\infty \frac{4+\cos(3x)}{x^4}$

$4+\cos(3x) \leq 5$ so $0 \leq \frac{4+\cos(3x)}{x^4} \leq \frac{5}{x^4}$

$\lim_{b\to\infty} \int_1^\infty \frac{5}{x^4} dx = \lim_{b\to\infty} \left[-\frac{5}{3}x^{-3}\right]_1^\infty =$

$\lim_{b\to\infty} \left[-\frac{5}{3b^3} - -\frac{5}{3(1)^3}\right] = \frac{5}{3}$

Since $\int_1^\infty \frac{5}{x^4}$ converges so does $\int_1^\infty \frac{4+\cos(3x)}{x^4} dx$ converges

6. $\int_0^{\frac{\pi}{4}} \frac{\cos(2x)}{\sqrt{\sin(2x)}} dx$ infinite discontinuity at $x=0$

$\lim_{b \to 0^+} \int_b^{\frac{\pi}{4}} \frac{\cos(2x)}{\sqrt{\sin 2x}}$ $\frac{1}{2} \int \frac{2\cos(2x)}{\sqrt{\sin(2x)}} dx =$

$u = \sin 2x$

$du = 2\cos 2x$

$\frac{1}{2} \int_{x=b}^{x=\frac{\pi}{4}} u^{-1/2} du = u^{1/2} = \sqrt{\sin(2x)}$

$\lim_{b \to 0^+} \left[\sqrt{\sin(2x)} \right]_b^{\frac{\pi}{4}} = \lim_{b \to 0} \left(\sqrt{\sin \frac{\pi}{2}} - \sqrt{\sin b} \right) = \boxed{1}$

7.

$\int_0^{\infty} x e^{-x} dx = \lim_{b \to \infty} \int_0^b x e^{-x} dx$ $u = x$ $dv = e^{-x}$

$du = 1dx$ $v = -e^{-x}$

$\int x e^{-x} = -xe^{-x} - \int -e^{-x} dx$

$\lim_{b \to \infty} \left[-xe^{-x} - e^{-x} \right]_0^b$

$\frac{-b}{e^b} - \frac{1}{e^b} + 0 + \frac{1}{e^0} = \boxed{1}$

8. $\int \frac{x-13}{x^2+4x-5}\,dx =$

$$\frac{A}{x+5} + \frac{B}{x-1} = \frac{x-13}{(x+5)(x-5)}$$

$$A(x-1) + B(x+5) = x-13$$

$x=1$

$6B = -12$

$B = -2$

$x = -5$

$-6B = -18$

$A = 3$

$$\int \left(\frac{3}{x+5} - \frac{2}{x-1}\right) dx$$

$$\boxed{3\ln(x+5) - 2\ln(x-1) + C}$$

E.C.

$$\int_e^{\infty} \frac{3}{x(\ln(x))^2}\,dx$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$\int \frac{3}{x(\ln x)^2}\,dx = 3\int u^{-2}\,du = -3u^{-1} = -\frac{3}{\ln x} + C$$

$$\lim_{b\to\infty}\left[-\frac{3}{\ln x}\right]_e^b = \lim_{b\to\infty}\left[-\frac{3}{\ln b} + \frac{3}{\ln e}\right] = \frac{3}{1} = \boxed{3}$$