

Section 2.2 ^{Calc BC} Limits involving infinity

End behavior Model

as $x \rightarrow \infty$ only the largest exponents of polynomial functions becomes significant

Example $\lim_{x \rightarrow \infty} \frac{3x^5 + x^4 + 2x^2 + 2}{4x^2 + 3x}$ behaves

the same as $\frac{3x^5}{4x^2} \rightarrow 3x^3$

We call this the end behavior model

Example Find the end behavior model of

$$f(x) = \frac{4x^3 + 2x^2 + 2}{5x^3 + 2x + 2} \Rightarrow$$

$$\text{End behavior} = \frac{4x^3}{5x^3} = \frac{4}{5}$$

When the limit as $x \rightarrow \infty$ or $x \rightarrow -\infty =$ a

constant or linear function the function has a horizontal asymptote at that point.

Example Find the horizontal asymptote

$$\text{for } g(x) = \frac{5x^2 + 7x + 2}{3x^2 - 5}$$

$$\lim_{x \rightarrow +\infty} = \frac{5x^2}{3x^2} = \frac{5}{3}$$

$$\lim_{x \rightarrow -\infty} = \frac{5}{3}$$

Horizontal asymptote ~~at~~ at $x = \frac{5}{3}$ on
both the left and right.

Example Find the horizontal asymptote,

$$y = \frac{2x+3}{|x|+1} \quad \lim_{x \rightarrow +\infty} = \frac{2x}{x} = 2$$

$$\lim_{x \rightarrow -\infty} = \frac{2x}{-x} = -2$$

Vertical Asymptotes

Ex Find the asymptotes for $f(x)$

$$f(x) = \frac{3x+2}{x-1}$$

$$\lim_{x \rightarrow \infty} f(x) = 3$$

$$\lim_{x \rightarrow -\infty} f(x) = 3$$

$$\lim_{x \rightarrow 1^+} f(x) = \infty$$

$$\lim_{x \rightarrow 1^-} f(x) = -\infty$$

Horizontal Asymptotes at $x=3$
on the left and the right

Vertical asymptote ~~up~~ on
the ~~left~~ of $x=1$ and
right

down on the left of $x=1$