

# Calc BC Section 3.1 Derivatives.

Pg 1/3

Warm up: Find the slope at  $x=2$

$$f(x) = x^2 - x \quad \text{use the def of instantaneous slope}$$

Notations for Deriv.

$$f', y', \frac{dy}{dx}, \frac{d}{dx} f(x)$$

Def of the Derivative

$$\text{A. } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \quad \text{or} \quad \text{B. } \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

Example: Find the derivative of

$$f(x) = x^2 + 4 \quad \text{at } a = 1 \quad \text{using def A}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+h)^2 + 4 - (x^2 + 4)}{h} =$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{x^2} + 2xh + h^2 + \cancel{4} - \cancel{x^2} - \cancel{4}}{h} =$$

$$f'(x) = \lim_{h \rightarrow 0} (2x + h) = 2x$$

$$f'(1) = 2(1) = \boxed{2}$$

Find the derivative of  $f(x) = \sqrt{x+1}$  at  $x=3$

using  $f'(a) = \lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$

$$a = 3$$

$$f'(3) = \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - \sqrt{3+1}}{x-3} = \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3}$$

$$= \lim_{x \rightarrow 3} \frac{\sqrt{x+1} - 2}{x-3} \cdot \frac{\sqrt{x+1} + 2}{\sqrt{x+1} + 2} =$$

$$= \lim_{x \rightarrow 3} \frac{x+1-4}{(x-3)(\sqrt{x+1}+2)} = \lim_{x \rightarrow 3} \frac{1}{\sqrt{x+1}+2}$$

$$= \frac{1}{\sqrt{4}+2} = \boxed{\frac{1}{4}}$$

### One Sided derivatives

Find the left hand derivative of

$$y = |x| \quad \text{at } x=0,$$

$$y = -x \quad \text{when } x < 0$$

$$\lim_{x \rightarrow 0^-} \frac{f(x) - f(a)}{x - a} = \lim_{x \rightarrow 0^-} \frac{-x - 0}{x - 0} = \lim_{x \rightarrow 0^-} \frac{-x}{x} = \boxed{-1}$$

