

Warm Up \rightarrow 1. Find y'

a) $y = \frac{1}{x^2} - x^2 \cos x$

b) $y = \frac{x^2}{1 + \cos x}$

 \rightarrow 2. Find the equation of the tangent and Normal to $y = 2 \cos x + x$ at $x = \frac{\pi}{4}$.

Ex 1 $y = 3u$, $u = 2x - 5$

y of $u = 3(2x - 5) = 6x - 15$

$\frac{dy}{du} = 3$ $\frac{du}{dx} = 2$ $\frac{d}{dx}(y \circ u) = 6$

Chain - Rule

$\frac{dy}{du} \cdot \frac{du}{dx} = \frac{dy}{dx}$

Ex 2

$y = u^2$ $u = 2x^2 + 3$ $y(4) =$

Find $\frac{dy}{dx}$ $\frac{dy}{du} = 2u$ $\frac{du}{dx} = 4x$
 $= 2(2x^2 + 3) = 4x^2 + 6$

$$x(t) = \sin(t^3 + 1) \quad \text{Find } \frac{dx}{dt}$$

Remember

$$\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt}$$

Let $u = t^3 + 1$ so $x(u) = \sin u$

$$\frac{du}{dt} = 3t^2 \quad \frac{dx}{du} = \cos u$$

$$\frac{dx}{dt} = \frac{dx}{du} \cdot \frac{du}{dt} = (\cos u)(3t^2)$$

$$\boxed{\frac{dx}{dt} = \cos(t^3 + 1)(3t^2)}$$

Chain Rule as inner/outer

~~AB~~
$$y = f(g(x))$$

$$y' = f'(g(x)) g'(x)$$

Ex Find y' if $y = \sec(x^2 + 4)$

\uparrow $\underbrace{\hspace{2em}}$
 outer inner

$$y' = \sec(x^2 + 4) \tan(x^2 + 4)$$

Chain Rule - Repeated Use

Pg. 3.

Ex

$$\frac{d}{dx} \left(\sin^3(x^2+3) \right) = \boxed{3 \sin^2(x^2+3) \cos(x^2+3) (2x)}$$

Ex

Parametric

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

Ex

$$x = 2 \cos t \quad y = 2 \sin t$$

Find the equation of the tangent to the curve at $x = \frac{\pi}{6}$

$$\frac{dx}{dt} = -2 \sin t \quad \frac{dy}{dt} = 2 \cos t$$

$$\begin{aligned} \frac{dx}{dt} \Big|_{t=\frac{\pi}{6}} &= -2 \sin \frac{\pi}{6} & \frac{dy}{dt} \Big|_{t=\frac{\pi}{6}} &= 2 \cos \frac{\pi}{6} \\ &= -2 \left(\frac{1}{2} \right) & &= 2 \left(\frac{\sqrt{3}}{2} \right) \\ &= -1 & &= \sqrt{3} \end{aligned}$$

$$\frac{dy}{dx} \Big|_{t=\frac{\pi}{6}} = \frac{\sqrt{3}}{-1} = -\sqrt{3} \quad \text{Point } \left(2 \cos \frac{\pi}{6}, 2 \sin \frac{\pi}{6} \right)$$

Eq tangent

$$\boxed{y - 2 = -\sqrt{3}(x - \sqrt{3})}$$

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