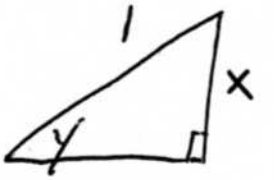
Calc BC Section 3.8

Derivatives of Inverse Trig Functions

Warm up

1.



Find $\sin y$, $\cos y$, $\tan y$

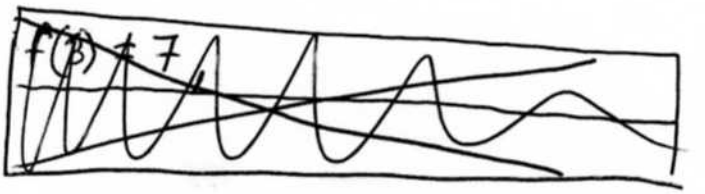
$X = f(s(x))$

$1 = f'(s(x))\,s'(x)$

$s'(x) = \frac{1}{f'(s(x))}$

2 If $f(2) = 3$ and $f^{-1}(x) = g(x)$ Given $f^{-1}(x) = g(x)$

$f'(3) = 4$



$f(2) = 3$

$f(3) = 5$

$f'(2) = 7$

$f'(3) = 8$

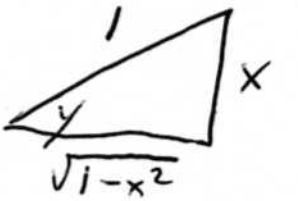
Find $g'(3) =$

3. Find $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$g'(3) = \frac{1}{f'(g(3))} = \frac{1}{f(3)} = \frac{1}{7}$

Derivative of Arc sin

$y = \sin^{-1}(x)$



$\sin y = x$

$(\cos y)(y') = 1$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

Apply the chain rule

If u is a diff function of x then

$$\frac{d}{dx}\sin^{-1}u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

Ex: $$\frac{d}{dx}\sin^{-1}(x^3) = \frac{1}{\sqrt{1-(x^3)^2}}\left(\frac{d}{dx}x^3\right)$$

$$= \frac{1}{\sqrt{1-x^6}}3x^2 = \frac{3x^2}{\sqrt{1-x^6}}$$

Derivative of Arc Tan

$y = \tan^{-1}(x)$ $\quad \tan y = x$

$\sec^2 y \frac{dy}{dx} = 1$

$y' = \frac{1}{\sec^2 y}$

$\sec y = \sqrt{x^2+1}$ $\quad y' = \frac{1}{x^2+1}$

$\sec^2 y = x^2+1$

Chain Rule $$\frac{d}{dx}\left(\tan^{-1}(u)\right) = \frac{1}{u^2+1}\frac{dy}{dx}$$

Example: $x(t) = \tan^{-1}\sqrt{t}$

find $x'(16)$

$$x'(t) = \frac{1}{(\sqrt{t})^2+1}\,\frac{1}{2}t^{-\frac{1}{2}} = \left(\frac{1}{t+1}\right)\left(\frac{1}{2\sqrt{t}}\right)$$

$$x'(16) = \frac{1}{16+1}\cdot\frac{1}{2(4)} = \frac{1}{136}$$

Arc sec

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}} \qquad \frac{d}{dx}\sec^{-1}u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$

Example

$$\frac{d}{dx}\sec^{-1}(4x^2) = \frac{1}{|4x^2|\sqrt{(4x^2)^2-1}}\cdot(8x)$$

$$= \frac{8x}{4x^2\sqrt{16x^4-1}} = \frac{2}{2x\sqrt{16x^4-1}}$$

Inverse Function – Inverse Co Function Identities

$$\cos^{-1}(x) = \frac{\pi}{2} - \sin^{-1}(x)$$

$$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$$

$$\csc^{-1}(x) = \frac{\pi}{2} - \sec^{-1}(x)$$

Calculator Conversion Identities

$$\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$$

$$\cot^{-1}(x) = \frac{\pi}{2} - \tan^{-1}(x)$$

$$\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$$

$$\frac{d}{dx}\cos^{-1}(u) = -\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

$$\frac{d}{dx}\cot^{-1}(u) = \frac{-1}{1+u^2}\frac{du}{dx}$$

$$\frac{d}{dx}\csc^{-1}(u) = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$$