Calc BC section 3.9

Warm up

1. $\frac{d}{dx} \sin^{-1}(x^3)$
2. $\frac{d}{dx} \cos^{-1}(3x^2)$
3. $\frac{d}{dx} \tan^{-1}(5x^2)$
4. $\frac{d}{dx} \sec(x^2)$

5. Graph e^x and $\frac{d}{dx} e^x$ on your calculator

Given: $\lim_{h \to 0} \frac{e^h - 1}{h} = 1$

$$\frac{d}{dx} e^x = \lim_{h \to 0} \frac{e^{x+h} - e^x}{h} = \lim_{h \to 0} \frac{e^x e^h - e^x}{h}$$

$$= e^x(1) = e^x$$

$$\frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx} e^u = e^u \frac{du}{dx}$$

$a^x = e^{x \ln a}$

$\Rightarrow \quad e^{x \ln a} = e^{\ln a^x} = a^x$

$$\frac{d}{dx} a^x = \frac{d}{dx} e^{x \ln a} = e^{x \ln a} \frac{d}{dx} x \ln a = a^x \ln a$$

Chain Rule

$$\frac{d}{dx} a^u = a^u \ln a \frac{du}{dx}$$

Ex Find where the slope of $f(x) = 2^x - 5x$ is zero

$$\frac{d}{dx}\left(2^x - 5x\right) = 2^x \ln 2 - 5$$

$$2^x \ln 2 - 5 = 0$$

$$2^x \ln 2 = 5$$

$$\ln\left(2^x\right) = \ln\left(\frac{5}{\ln 2}\right)$$

$$x \ln 2 = \ln\left(\frac{5}{\ln 2}\right)$$

$$x = \frac{\ln\left(\frac{5}{\ln 2}\right)}{\ln 2} = 2.851$$

$f'(x) = 0$ when $x = 2.851$

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Deriv. of $\ln x$

$y = \ln x$

$e^y = x$

$\frac{d}{dx}(e^y) = \frac{d}{dx}(x)$

$e^y y' = 1$

$y' = \frac{1}{e^y} = \frac{1}{x}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$ $\quad$ $\frac{d}{dx} \ln u = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}$

Ex $\quad \frac{d}{dx}[x \ln x] = x \frac{1}{x} + \ln x \cdot 1$

$= \boxed{1 + \ln x}$

~~Logarithmic Differentiation~~

Ex $\quad f(x) = \ln \sqrt{x+1}$

Find $f'(x)$

$f(x) = \ln (x+1)^{1/2}$

$f(x) = \frac{1}{2} \ln (x+1)$

$f'(x) = \frac{1}{2}\left(\frac{1}{x+1}\right) = \frac{1}{2(x+1)}$

Ex $f(x) = \ln \dfrac{x(x^2+1)^2}{\sqrt{2x^3-1}}$ Find $f'(x)$

using log rules

$$f(x) = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$$

$$f'(x) = \frac{1}{x} + 2\frac{1}{x^2+1}(2x) - \frac{1}{2}\frac{1}{(2x^3-1)}\cdot 6x^2$$

$$= \boxed{\frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}}$$

Logarithmic D.M.

$$y = \frac{(x-2)^2}{\sqrt{x^2+1}}$$ Find y'

$$\ln y = \ln \frac{(x-2)^2}{\sqrt{x^2+1}}$$

$$\ln y = 2\ln(x-2) - \frac{1}{2}\ln(x^2+1)$$

$$\frac{y'}{y} = \frac{2}{x-2} - \frac{1}{2}\frac{2x}{x^2+1} \Rightarrow y' = y\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

$$y' = \frac{(x-2)^2}{\sqrt{x^2+1}}\left[\frac{2}{x-2} - \frac{x}{x^2+1}\right]$$

Derivative of $\log_a x$

$$\log_a x = \frac{\ln x}{\ln a}$$

$$\frac{d}{dx} \log_a x = \frac{d}{dx}\left(\frac{\ln x}{\ln a}\right)$$

$$= \frac{1}{x} \cdot \frac{1}{\ln a} = \frac{1}{x \ln a}$$

Chain Rule

$$\frac{d}{dx} \log_a u = \frac{1}{u \ln a} \frac{du}{dx}$$