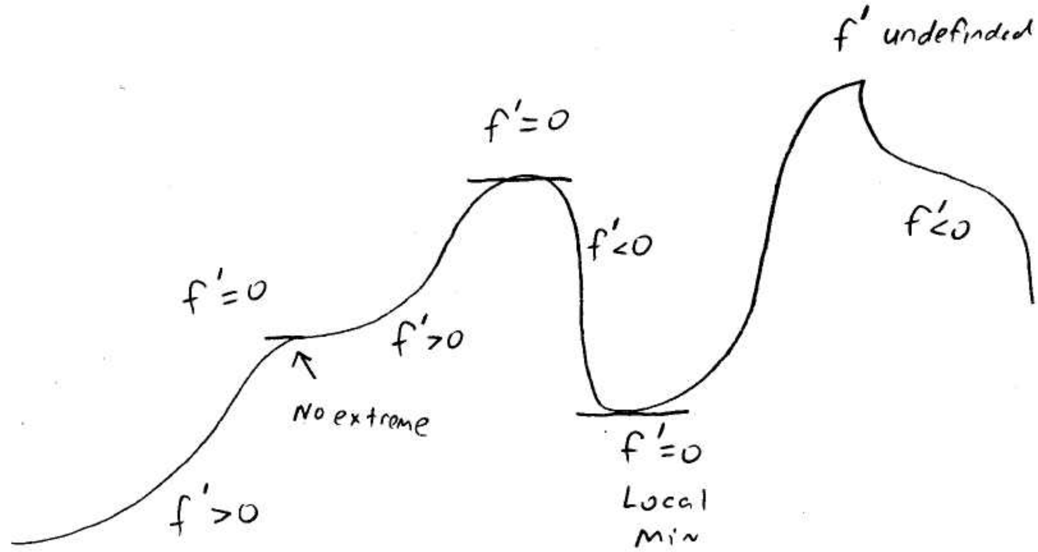
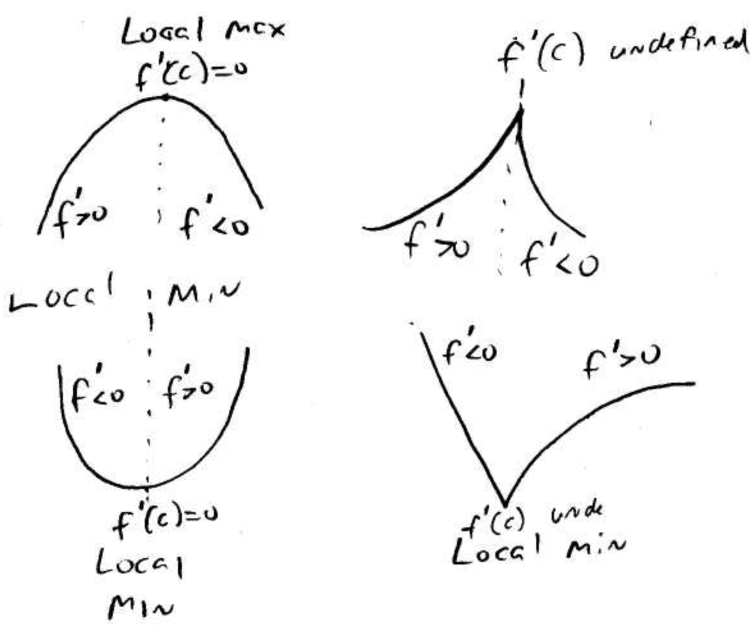
Calc BC Section 4.3 Pg 1 of

Connecting f' and f'' with the Graph of f



First Derivative Test



f' changes sign from (+) to (−)

f' changes sign from (−) to (+)

Look at book for other drawings

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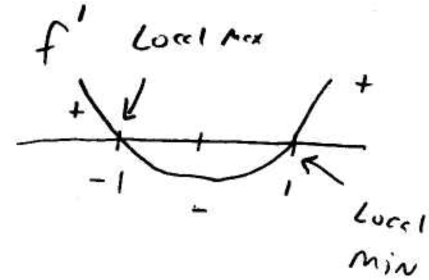
Example 1

find the local extrema using the 1st derivative test

a)

$$f(x) = 2x^3 - 6x + 7$$

$$f'(x) = 6x^2 + 6$$



$$6x^2 - 6 = 0$$

$$6(x-1)(x+1) = 0$$

$$x = 1 \quad x = -1$$

$$f(1) = 2($$

$$f(-1) = 2(-1) - 6(-1) + 7$$

$$= -2 + 6 + 7 = 11$$

Local max of 11 @ $x = -1$

b/c f' changes from (+) to (-).

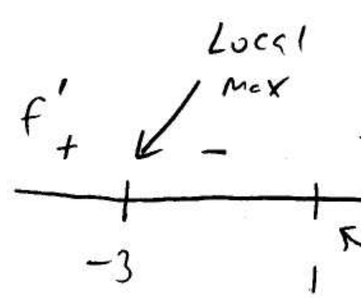
b)

$$g(x) = (x^2 - 3)e^x$$

$$g'(x) = (x^2 - 3)e^x + e^x(2x)$$

$$= e^x(x^2 + 2x - 3)$$

$$g'(x) = e^x(x+3)(x-1)$$



Max of $g(-3)$ at $x = -3$

b/c f' changes fron (+) to neg

min of $g(1)$ at $x = 1$

b/c f' chances fron (-) to (+)

Concavity

Graph concave up when f' is increasing
concave down when f' is decreasing

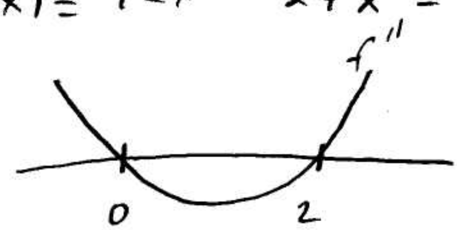
up down

Point of inflection: A place where the graph of a function has a tangent line and where the concavity changes.

Example: Find the points of inflection and determine the regions of concavity of

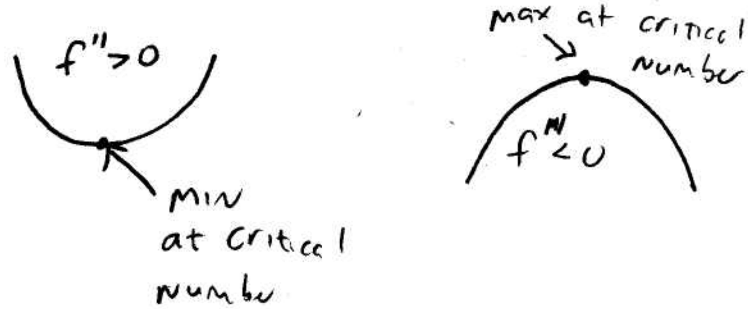
$$f(x) = x^4 - 4x^3$$
$$f'(x) = 4x^3 - 12x^2$$
$$f''(x) = 12x^2 - 24x = 12x(x-2)$$

$x=0 \quad x=2$



f is concave up for $x<0$ or $x>2$
f is concave down for $0<x<2$
points of inflection at $x=0$ and $x=2$

Second Derivative Test



Example Find the x-value of the relative extrema and points of inflection for $f(x) = -3x^5 + 5x^3$

$$f'(x) = -15x^4 + 15x^2$$
$$= -15x^2(x^2-1)$$
$$= -15x^2(x+1)(x-1)$$

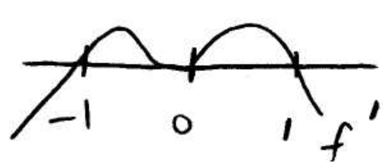
Critical #'s at $x=0$, $x=-1$, $x=1$

$$f''(x) = -60x^3 + 30x = \underbrace{-30x(2x^2-1)}_{\text{Double root}}$$

$f''(-1) = 30 > 0$ concave up $\rightarrow$ rel MIN @ $X=-1$

$f''(1) = -30 < 0$ concave down $\rightarrow$ rel Max @ $X=1$

$f''(0) = 0$ Test Fails so use the 1st deriv. test



f' does not change signs so No

points of inflection @ $x=-\sqrt{\frac{1}{2}}$, $x=\sqrt{\frac{1}{2}}$, $x=0$