Calc BC Linearization / Newtons Method
differentials

$$y - y_1 = m(x - x_1)$$

Linearization of f at f(a)

$y = L(x)$

$y_1 = f(a)$

$m = f'(a)$

$x_1 = a$

$$L(x) - f(a) = f'(a)(x - a)$$

$$\longrightarrow \quad L(x) = f(a) + f'(a)(x-a)$$

Example: Find the linearization of $f(x) = \sqrt{4+x}$ at $a = 0$, and use it to approximate $\sqrt{4.1}$ w/o a c

$f(0) = \sqrt{4} = 2$

$f'(x) = \frac{1}{2}(4+x)^{-1/2} = \frac{1}{2\sqrt{4+x}}$

$f'(0) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$

$L(x) = 2 + \frac{1}{4}(x-0) = 2 + \frac{1}{4}(x)$

$\sqrt{4.1} = f(.1) \qquad L(.1) = 2 + \frac{1}{4}(.1) = 2.025$

Find the linearization $y = \sin x$ at $x = \pi$

use it to estimate $\sin(3)$

$f(\pi) = 0$

$f'(x) = \cos x$

$f'(\pi) = -1$

$L(x) = 0 + -1(x - \pi) = -x + \pi$

$\sin(3) = -3 + \pi = -3 + 3.14159 \approx 0.14159$

Compare to calculator

$.14159 - .14112 = .0005$

Use linear approximations to estimate

the $\sqrt{145}$ $\qquad \sqrt{144} = 12$ so

center at 144

$f(x) = \sqrt{x}$ $\qquad f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$ $\qquad f'(144$

$L(x) = 12 + \frac{1}{24}(x - 144)$

$L(145) = 12 + \frac{1}{24}(1) = 12\frac{1}{24} = 12.0417$

$\sqrt{145} = 12.0416$

Newton's Method

$$X_{n+1} = X_n - \frac{f(x_n)}{f'(x_n)}$$

Example use newtons method to solve

$$x^3 - 2x + 2 = 0$$

$f(x) = x^3 - 2x + 2 \qquad f'(x) = 3x^2 - 2$

$X_{n+1} = X_n - \frac{x^3 - 2x + 2}{3x^2 - 2}$ Guess $x_0 = -1$

$= -1$

use the calculator

$-1 \to X$ | $Y_1 = x^3 - 2x + 2$

$X - \frac{Y_1}{Y_2} \to X$ | $Y_2 = \text{nderiv}(Y_1, x, x)$

Differentials

$\frac{dy}{dx} = f'(x) \qquad dy = f'(x)\,dx$

Example Find the differential dy and evaluate it for $x = 2$ and $dx = .02$

$y = 3x^2 + 7x \qquad dy = (6x + 7)\,dx$

when $x = 2$ and $dx = .02$

$dy = (12 + 7)(.02) = (19)(.02) = 0.38$

Example of using differentials to estimate % error.

How accurately do you need to measure the radius of a sphere to get the volume within 3%.

$$V = \frac{4}{3}\pi r^3 \qquad |\Delta V| = \frac{3}{100}\left(\frac{4}{3}\pi r^3\right)$$

$$|\Delta V| = \frac{\pi r^3}{25}$$

$$dV = 4\pi r^2 dr$$

Replace dV with $|\Delta V|$

$$|4\pi r^2 dr| = \frac{\pi r^3}{25}$$

$$|dr| = \frac{\cancel{\pi} r^{\cancel{3}}}{25} \cdot \frac{1}{4\cancel{\pi} \cancel{r^2}}$$

$$|dr| = \frac{r}{100} = 0.01r = 1\% \text{ of } r$$

You must measure r to within 1% to get a 3% accuracy of the volume