Calc BC Section 5.3

Rules for Definite Integrals

1. $\int_b^a f(x)\,dx = -\int_b^a f(x)\,dx$

2. $\int_a^a f(x)\,dx = 0$

3. $\int_a^b K f(x)\,dx = K \int_a^b f(x)\,dx$ for K = constant

4. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

5. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

6. $\min f \cdot (a-b) \leq \int_a^b f(x)\,dx \leq \max f \, (a-b)$

7. If $f(x) \geq g(x)$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$

   If $f(x) \geq 0$ on $[a, b] \Rightarrow \int_a^b f(x)\,dx \geq 0$

Example 1

If $\int_{-2}^{1} f(x) = 2$ $\quad \int_{1}^{3} f(x) = -3$ $\quad \int_{-2}^{1} g(x) = 5$

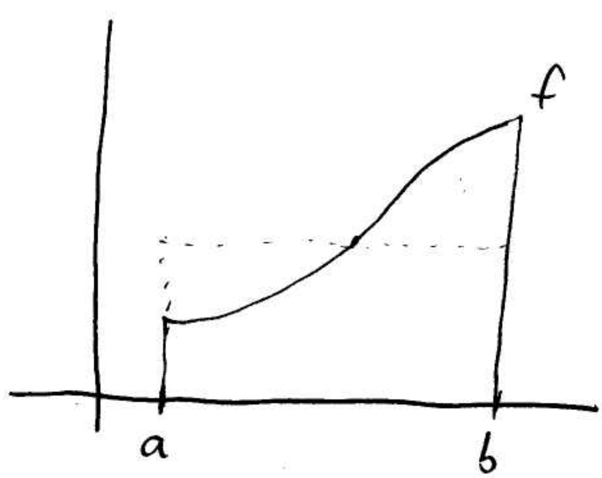
a) find $\int_{-2}^{1} 3f(x)\,dx$

b) find $\int_{1}^{-2} f(x)\,dx$

c) find $\int_{-2}^{1} \left[f(x) + g(x)\right] dx$

d) find $\int_{-2}^{3} f(x)\,dx$

Average value of a Function



$(\text{avg-}f)(b-a) = \int_a^b f(x)\,d.$

$\text{avg-}f = \frac{1}{b-a} \int_a^b f(x)\,dx$

Example 2

Find the average value of $f(x) = x^3 - 2x$ on the interval 1 to 4.

$$\text{average value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{4-1}\int_1^4 (x^3-2x)\,dx \quad \text{use fnInt}$$

$$= \left(\frac{1}{3}\right)(48.75) = 16.25$$

Fundamental Theorem of Calculus (Part Deux

If $F(x)$ is the anti derivative of $f(x)$

then $\star \int_a^b f(x)\,dx = F(b) - F(a)$

Example

$$\int_0^2 2x\,dx \quad \left[x^2\right]_0^2 = (2)^2 - 0^2 =$$

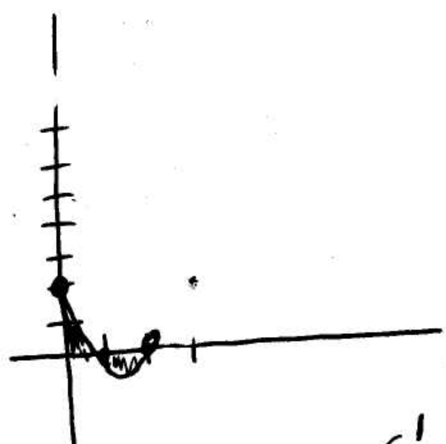
Example

Find the area between the curve and the x-axis on the interval

$$y = x^2 - 3x + 2 \qquad \text{on} \qquad [0, 2]$$

$$(x-2)(x-1)$$

$$y(0) = 2$$



$$\text{Area} = \int_0^1 (x^2 - 3x + 2) + -\int_1^2 (x^2 - 3x + 2)$$

$$\text{Area} = \left[\frac{x^3}{3} - \frac{3}{2}x^2 + 2x\right]_0^1 - \left[\frac{x^3}{3} - \frac{3}{2}x^2 + 2x\right]_1^2$$

$$\left(\frac{1}{3} - \frac{3}{2} + 2\right) - \left[\frac{8}{3} - \frac{3}{2}(4) + 4 - \left(\frac{1}{3} - \frac{3}{2} + 2\right.\right.$$

$$\frac{1}{3} - \frac{3}{2} + 2 - \frac{8}{3} + 6 - 4 + \frac{1}{3} - \frac{3}{2} + 2$$

$$-\frac{6}{3} - \frac{6}{2} + 6 = -2 - 3 + 6 = \boxed{1}$$