Integration by Parts Calc BC 6.3 Notes

$$\frac{d}{dx}[uv] = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\frac{d}{dx}[uv] = uv' + vu'$$

$uv = \int uv'\,dx + \int vu'\,dx$ Integrate both sides

$uv = \int u\,dv + \int v\,du$ rewrite to solve for $\int u\,dv$

$$\int u\,dv = uv - \int v\,du$$

Ex 1 $\int x e^x\,dx$ Let $u = x$, $du = 1\,dx$; $dv = e^x\,dx$, $v = e^x$

$$\int xe^x\,dx = xe^x - \int e^x\,dx$$
$$= xe^x - e^x + C$$

Guidelines for Integration by parts

1. Let dv be the most complicated part of the integral that fits a basic integration rule

2. Let u be the portion of the integrand whose derivative is simpler than u.

Ex 2

u

$$\int (3-5x)\cos(4x)\,dx =$$

$u = 3-5x$

$du = -5\,dx$

$dV = \cos(4x)\,dx$

$V = \frac{1}{4}\sin(4x)$

$$\int u\,dv = uv - \int v\,du$$

$$= (3-5x)\left(\tfrac{1}{4}\sin(4x)\right) - \int \tfrac{1}{4}\sin(4x)(-5\,dx)$$

$$= (3-5x)\left(\tfrac{1}{4}\sin 4x\right) + \tfrac{5}{4}\int \sin(4x)\,dx$$

$$= (3-5x)\left(\tfrac{1}{4}\sin 4x\right) + \tfrac{5}{4}\cdot -\tfrac{1}{4}\cos 4x + c$$

$$= \frac{3-5x}{4}\sin(4x) - \frac{5}{16}\cos(4x) + C$$

Ex 3

$$\int (7-3x)e^{6x}\,dx$$

$u = 7-3x$ $\quad dV = e^{6x}\,dx$

$du = -3\,dx$ $\quad V = \frac{1}{6}e^{6x}$

$$\int u\,dv = uv - \int v\,du$$

$$= (7-3x)\left(\tfrac{1}{6}e^{6x}\right) - \int \tfrac{1}{6}e^{6x}(-3\,dx)$$

$$= (7-3x)\left(\tfrac{1}{6}e^{6x}\right) + \tfrac{1}{2}\int e^{6x}\,dx$$

$$= \boxed{\frac{(7-3x)e^{6x}}{6} + \frac{e^{6x}}{12} + C}$$

Tabular Method

Ex 4

$$\int x^2 \cos 3x \, dx$$

Alt Signs		u and its Derivatives	v' and its anti derivatives
$+$	$\longrightarrow$	x^2	$\cos 3x$
$-$	$\longrightarrow$	$2x$	$\frac{1}{3}\sin 3x$
$+$	$\longrightarrow$	2	$-\frac{1}{9}\cos 3x$
$-$		0	$-\frac{1}{27}\sin 3x$

$$\int x^2 \cos 3x = \frac{1}{3}x^2 \sin 3x + \frac{2x}{9}\cos 3x - \frac{2}{27}\sin 3x + C$$

Ex 5

$$\int \ln x \, dx$$

$$\int u \, dv = uv - \int v \, du \qquad u = \ln x \quad dv = dx$$

$$du = \frac{1}{x}dx \quad v = x$$

$$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} dx$$

$$= x \ln x - \int 1 \, dx$$

$$= x \ln x - x + C$$

$$\int e^x \sin x \, dx$$

$u = e^x \qquad dV = \sin x$

$du = e^x dx \qquad V = -\cos x$

$$\int e^x \sin x \, dx = -e^x \cos x + \int + e^x \cos x \, dx$$

$u = e^x \qquad dv = \cos x$

$du = e^x dx \qquad v = \sin x$

$$\int e^x \sin x dx = -e^x \cos x + e^x \sin x - \int e^x \sin x \, dx$$

Add to both sides

$$2\int e^x \sin x dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x dx = \frac{-e^x \cos x + e^x \sin x}{2}$$

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