Integrals as Net Change

Example: A particle is moving along the x-axis according to the velocity function

$v(t) = 6\sin 3t \quad 0 \le t \le \frac{\pi}{2}$

Find the total distance traveled by the particle and its final position if $s(0) = 4$.

$$s(t) = \int 6\sin 3t\, dt$$

$$= -2\cos 3t + C$$

$$s(0) = 4 = -2\cos(3(0)) + C$$

$$-2 = -1 + C$$

$$C = -1$$

$$s(t) = -2\cos 3t - 1.$$

$$s\left(\tfrac{\pi}{2}\right) = -2\cos\tfrac{\pi}{2} - 1 = -2(0) - 1 = -1$$

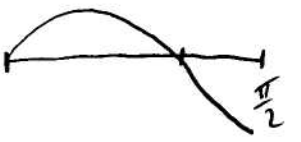
To find the total distance traveled we need to find the pos and neg velocities

$6\sin 3t = 0$

$\sin 3t = 0$

$3t = 0 \qquad 3t = \pi$

$t = 0 \qquad t = \frac{\pi}{3}$



Total Distance =

$$\int_0^{\frac{\pi}{3}} 6\sin 3t\,dt + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 6\sin 2t\,dt$$

$$\left[-2\cos 3t\right]_0^{\frac{\pi}{3}} - \left[-2\cos 2t\right]_{\frac{\pi}{3}}^{\frac{\pi}{2}}$$

$$-2\cos\pi - -2\cos 0 + 2\cos\frac{3\pi}{2} - 2\cos\pi$$

$$2 + 2 - -2 = \boxed{6}$$

Or use $\text{fnInt}\left(\text{abs}(Y_1), X, 0, \frac{\pi}{2}\right)$

Example 2 A car moving at 6 miles per hour accelerates at the rate of 4 miles per hour per sec. for 10 seconds,

a) How fast was the car going after 10 seconds

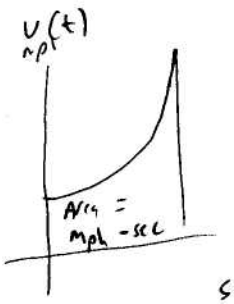
b) How far had the car traveled.

a) $a(t) = 4t$ $\quad v(t) = \int_0^{10} 4t\,dt = \left(2t^2\right)_0^{10}$ + initial

$v(t) = 200$ mph + 6 mph

$= \boxed{206 \text{ mph}}$

b) Distance traveled =



$$\int_0^{10} |v(t)|\,dt = \int_{0\text{ sec}}^{10\text{ sec}} 6 + 2t^2\,dt$$

$$= \left(6t + \frac{2}{3}t^3\right)_0^{10} = 60 + \frac{2}{3}(1000)$$

$$= 60 + \frac{2000}{3}$$

$$= 726.66 \text{ mph - sec}$$

$$726.66 \frac{\text{mi}}{\text{hr}} \cdot \text{sec} \cdot \frac{1\text{ hr}}{3600\text{ sec}} = 0.202 \text{ mile}$$

Work

$W = Fd$ units Foot-Pound or Newton meter (Joule)

Hooke's Law for Springs

$$F = kd$$

A force of 750 pounds compresses a spring 3 inches from its natural length of 15 inches. Find the work done in compressing the spring 6 inches.

$$750 = k(3)$$

$$k = 250$$

$$\text{Work} = \int_0^6 250x\,dx = \left[125x^2\right]_0^6 = 4500 \text{ in-pounds}$$