

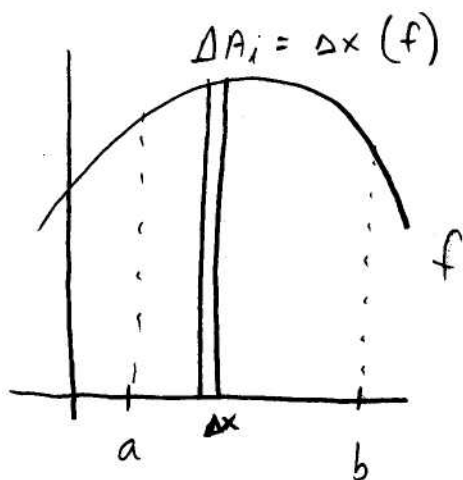
Calc BC

Section 7.2

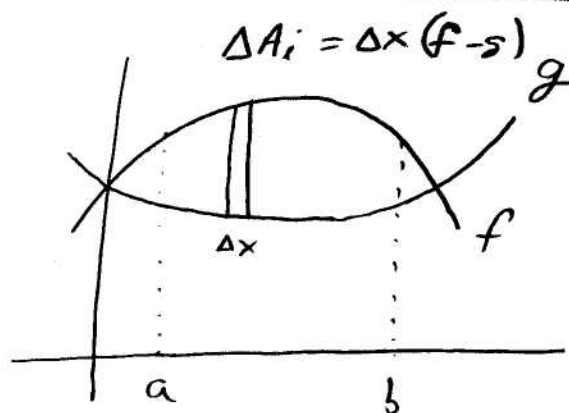
Area between two
curves

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Warm-up: Find the area under $y = x^2 + 2$
from $x = 0$ to $x = 4$



$$\text{Area} = \int_a^b f \, dx$$



$$\text{Area} = \int_a^b (f - g) \, dx$$

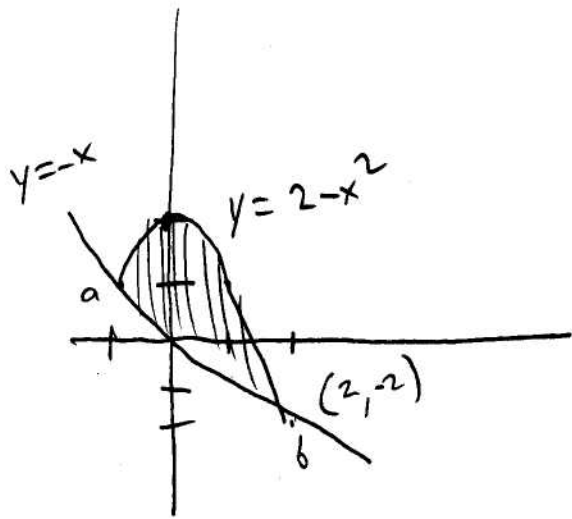
To find the area between two curves on an interval, simply take the integral of the top curve minus the bottom curve

If $f(x) > g(x)$ on the interval $[a, b]$

$$\text{Area} = \int_a^b (f(x) - g(x)) \, dx$$

Example: Find the area bounded by
 $y = -x$ and $y = 2 - x^2$

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Graph both curves
and find where they cross

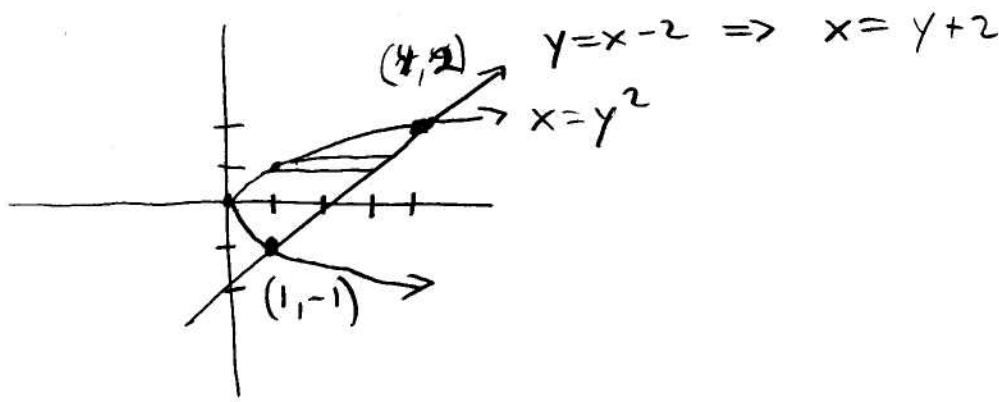
$$\begin{aligned} -x &= 2 - x^2 \\ x^2 - x - 2 &= 0 \\ (x-2)(x+1) &= 0 \\ x &= 2, x = -1 \end{aligned}$$

$$\int_{-1}^2 (2 - x^2 - (-x)) dx = \int_{-1}^2 (2 - x^2 + x) dx$$

$$\begin{aligned} &= \left[2x - \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^2 = \left(4 - \frac{8}{3} + 2 \right) - \left(-2 + \frac{1}{3} + \frac{1}{2} \right) \\ &= 4 - \frac{8}{3} + 2 + 2 - \frac{1}{3} - \frac{1}{2} \\ &= 8 - \frac{9}{3} - \frac{1}{2} = \boxed{4\frac{1}{2}} \end{aligned}$$

Example Horizontal Slices

Find the area enclosed by $x = y^2$ and $y = x - 2$



Horizontal slices give us one interval.

$$\Delta A_i = (x_2 - x_1)(\Delta y)$$

big - little or right - left

$$A = \int_{-1}^2 [(y+2) - y^2] dy$$

$$\left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^2$$

4.5

$$x = y^2$$

$$x = y + 2$$

$$y^2 = y + 2$$

$$y^2 - y - 2 = 0$$

$$(y+1)(y-2) = 0$$

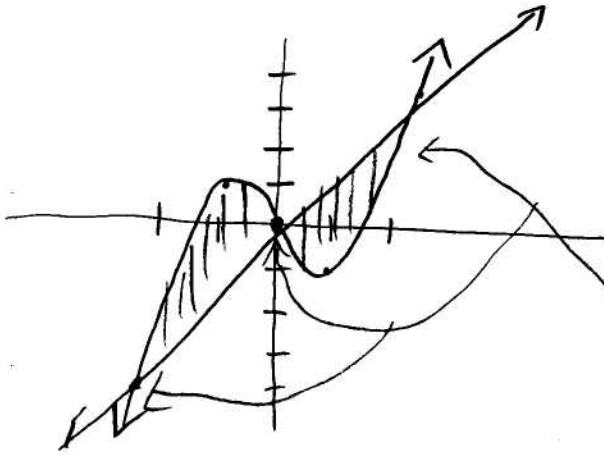
$$y = -1 \quad y = 2$$

$$y = x^3 - 2x$$

$$y = 2x$$

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Find the area between the two curves.



x	$x^3 - 2x$
0	0
1	-1
2	4
-1	1
-2	-4

x	2x
0	0
2	4
-2	-4

$$x^3 - 2x = 2x$$

$$x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x = 0 \quad x = 2 \quad x = -2$$

$$\int_{-2}^0 (x^3 - 2x) - 2x \, dx$$

$$+ \int_0^2 2x - (x^3 - 2x) \, dx$$

$$\int_{-2}^0 (x^3 - 4x) \, dx + \int_0^2 (4x - x^3) \, dx$$

$$\left[\frac{x^4}{4} - 2x^2 \right]_{-2}^0 + \left[2x^2 - \frac{x^4}{4} \right]_0^2$$

$$4 + 4 = \boxed{8}$$