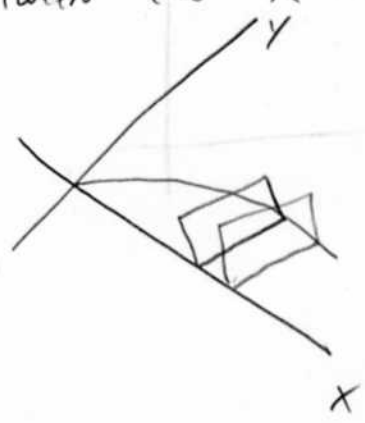


Volumes of Solids with Known Cross Section

$$V = \int_a^b A(x) dx \quad \text{or} \quad \int_a^b A(y) dy$$

Ex A solid is generated by placing ^{the sides of} squares between the x-axis and the function $y = \sqrt{x}$.



From $x=0$ to $x=9$

Find the volume of the solid

$$V = \int_a^b A_x dx$$

$$A_x = s^2 \quad \text{for a square}$$

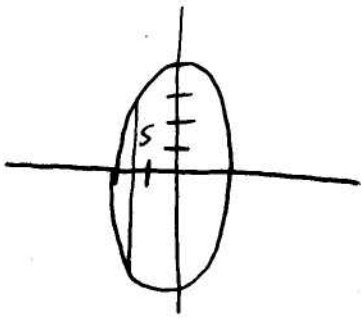
$$s = \sqrt{x} \quad s^2 = x$$

$$A_x = x$$

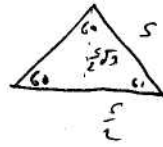
$$\int_0^9 x dx = \left[\frac{x^2}{2} \right]_0^9 = \boxed{\frac{81}{2}}$$

Ex A solid is formed by placing equal-sized Δ 's perpendicular to the x-axis between

$y = x^2 - 4$ and $y = 4 - x^2$ from $x = -2$ to $x = 2$. Find the volume of the solid



$$A_x = \left(\frac{s}{2}\right)^2 \sqrt{3}$$



$$A_x = \frac{s^2 \sqrt{3}}{4}$$

$$s = (4-x^2) - (x^2-4) = 8-2x^2$$

$$A_x = \frac{(8-2x^2)^2 \sqrt{3}}{4} = \frac{\sqrt{3}}{4} (64 - 32x^2 + 4x^4)$$

Use
CSIC
to
solve

Symmetry

$$A_{\text{area}} = 2 \int_0^2 \frac{\sqrt{3}}{4} (64 - 32x^2 + 4x^4) dx$$

$$= \frac{2\sqrt{3}}{4} \left[64x - \frac{32}{3}x^3 + \frac{4}{5}x^5 \right]_0^2$$

$$= 2\sqrt{3} \left[32 - \frac{64}{3} + \frac{32}{5} \right]$$

$$= 2\sqrt{3} \left[\frac{256}{15} \right] = \frac{512\sqrt{3}}{15}$$

$y = x^2$ U

A solid is to be generated by placing semi-circles the diameters of
perpendicular to the y-axis from $x = -\sqrt{y}$ to $x = \sqrt{y}$.

Find the area of the solid from $y = 0$ to $y = 4$.

$$A_y = \frac{1}{2} \pi r^2$$

$$r = \sqrt{y}$$

$$A_{\text{area}} = \int_0^4 \frac{1}{2} \pi y dy =$$

$$A_y = \frac{1}{2} \pi y$$

$$\frac{1}{2} \pi \left[\frac{y^2}{2} \right]_0^4 = \frac{1}{2} \pi \left[\frac{16}{2} - 0 \right] = 4\pi$$