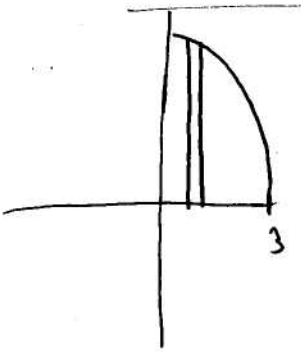
Volumes of surfaces of revolution

Find the volume of the solid generated by revolving the area around the x-axis

Area $y = 9 - x^2$ from $x = 0$ to $x = 3$



$$A_x = \pi r^2$$

$$V = \int_a^b A_x \, dx = \int_0^3 \pi r^2 \, dx$$

$$r = 9 - x^2$$

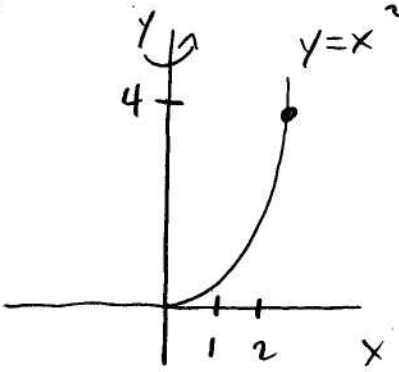
$$r^2 = (9 - x^2)^2 = 81 - 18x^2 + x^4$$

$$V = \pi \int_0^3 (9 - x^2)^2 \, dx = \frac{648\pi}{5} \, u^3$$

Around the y-axis

Find the volume created by rotating the area around the y-axis

Area bounded by $y = 0$ $y = x^2$



$y = x^2 \Rightarrow x = \sqrt{y}$

$$V = \int A_y \, dy$$

$$A_y = \pi r_y^2 \qquad r_y = \sqrt{y}$$

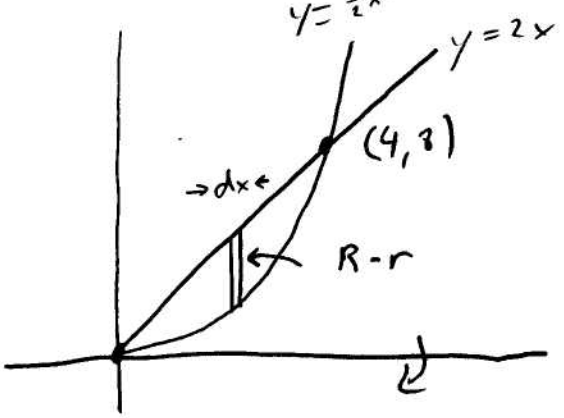
$$A_y = \pi y$$

$$V = \pi \int_{y=0}^{y=4} y \, dy = \left[\pi \frac{y^2}{2} \right]_0^4 = \boxed{8\pi}$$

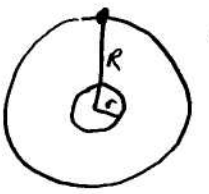
Washer - Method

Find the volume of the area rotated around the x-axis

Area $y = \frac{1}{2}x^2$ $y = 2x$



$$A_x = \pi (R^2 - r^2)$$



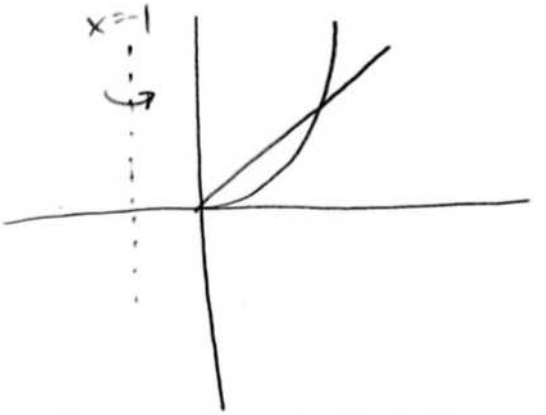
$R = 2x$

$r = \frac{1}{2}x^2$

$$\pi \int_0^4 \left[(2x)^2 - \left(\frac{1}{2}x^2 \right)^2 \right] dx = \frac{512\pi}{15}$$

Find the volume of the solid obtained by rotating the area around the line $x=-1$

Area $y=x$ $y=x^2$



$$V=\int A_y\,dy$$

$$A_y=\pi R^2-\pi r^2$$

$$R=\sqrt{y}--1 \qquad r=y--1$$

$$R^2=y+2\sqrt{y}+1 \qquad r^2=y^2+2y+1$$

$$V=\pi\int_0^1\left(y+2\sqrt{y}+1-y^2-2y-1\right)dy$$

$$V=\pi\int_0^1\left(2y^{1/2}-y^2-y\right)dy=\boxed{\frac{\pi}{2}}$$